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RESEARCH ARTICLE

**Engineering**

# Analysis of the influence of air quality on the performance of the Boeing 737-800 evaluated during takeoff at El Dorado Airport

## Análisis de la influencia de la calidad del aire en el rendimiento del Boeing 737-800 evaluado durante el despegue en el aeropuerto de El Dorado

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**Abstract.** This paper analyzes how the Bogotá's air quality and pollutant levels affects the Boeing 737-800 take-off performance in terms of the available runway distances and maximum take-off weight, which are two of the most restrictive factors in airlines' operation. The airport runway's physical characteristics, geographical position, surroundings, and atmospheric conditions affect the maximum aircraft take-off weight. This study examines: (a) the influence of pollutants in Bogotá's air quality and its subsequent effects in air density; (b) the maximum available polluting agents and relative humidity measures in this city; (c) the maximum take-off weight of the aircraft with and without air degradation due to atmospheric considerations. Software called "Aircraft Take-Off Software" has been developed to determine the B737-800 most limiting take-off weight of in specific geographical and atmospheric conditions. This study reflects the effect of high populated cities pollution on the aircraft operational performance which analysis has demonstrated that the presence of high carbon dioxide concentrations in the air increase the B737-800 payload capacity taking off from Bogotá but in the other hand, relative humidity variation decreases this capacity drastically, showing that considering Bogotá's relative humidity and pollutant levels, the most restrictive weight situation for the take-off phase is the "Climb Limit Take-Off Weight"; however, as it is an aerodynamic study of the pollutants influence in engine combustion the structural integrity was not considered and for further investigation it should be considered to determine more accurately the most restrictive take-off weight.

**Keywords:** Aircraft performance, air quality, carbon dioxide, relative humidity, density, take-off weight, pollutants, payload, software development.

## Resumen

Este documento analiza cómo la calidad del aire de Bogotá y los niveles de contaminantes afectan el rendimiento de despegue del Boeing 737-800 en términos de las distancias de pista disponibles y el Peso Máximo de Despegue, los cuales son dos de los factores más restrictivos en la operación de las aerolíneas. Las características físicas de la pista del aeropuerto, su situación geográfica, el entorno y las condiciones atmosféricas influyen en el peso máximo de despegue de las aeronaves. Este estudio examina: (a) la influencia de los contaminantes en la calidad del aire de Bogotá y sus efectos subsiguientes en la densidad del aire; (b) los agentes contaminantes máximos disponibles y las medidas de humedad relativa en esta ciudad; (c) el peso máximo de despegue de la aeronave con y sin degradación del aire debido a consideraciones atmosféricas. Se ha desarrollado un programa informático denominado "Aircraft Take-Off Software" para determinar el peso de despegue más restrictivo del B737-800 en condiciones geográficas y atmosféricas específicas. Este estudio refleja el efecto de la contaminación de ciudades altamente pobladas sobre el desempeño operacional de la aeronave, cuyo análisis ha demostrado que la presencia de altas concentraciones de dióxido de carbono en el aire aumenta la capacidad de carga útil del B737-800 despegando de Bogotá, pero por otro lado, la variación de la humedad relativa disminuye drásticamente esta capacidad, demostrando que considerando la humedad relativa y los niveles de contaminantes de Bogotá, la situación de peso más restrictiva para la fase de despegue es el "Climb Limit Take-Off Weight"; Sin embargo, por tratarse de un estudio aerodinámico de la influencia de los contaminantes en la combustión del motor, no se consideró la integridad estructural, la cual debería ser considerada en investigaciones posteriores para determinar con mayor precisión el peso más restrictivo de despegue.

**Palabras clave:** Rendimiento del avión, calidad del aire, dióxido de carbono, humedad relativa, densidad, peso al despegue, contaminantes, carga útil, desarrollo de software.

## 1 | INTRODUCTION

Pollution in the world has achieved the highest levels on the last years [1]. Since the time of the industrial revolution, the humanity has influenced the environmental degradation significantly due to all the combustion process that has developed since that time [2]. These generate interest in the effects of these industrial processes and to investigate in which parameters the atmosphere is affected. In recent years, an increase in pollutant gases and particulate matter has been detected in large cities around the world, and Bogotá is not an exception [3]. In this city several environmental alerts has been decreed because of an increment on the concentration of pollutants, which lead to taking effective preventive measures in order to improve air quality [4]. Environmental pollution has gained great prominence and has set off alarms in a notable number of countries, making them take decisions and actions that lead them to an environmental conscience, thus improving the quality of life of the inhabitants, and their environment, [5] in Colombia the governmental institute in charge to measure the air quality is the IDEAM [6].

The aeronautical industry produces close to 2% of all the carbon dioxide emissions generated by human activities [7]. Thus, in this industry, there is a concern on the effects of the air quality due to pollution on the performance of an aircraft. Therefore studies have been done like the investigation “Predictions of Operational degradation of the Fan Stage of an Aircraft Engine Due to Particulate Ingestion” [8]. Parallel to that, there are no precedents studies that show directly the impacts of pollution gases present in the atmosphere on the aerodynamic performance of an aircraft.

One of the main variables during the operation that can be affected is the Maximum Take-Off Weight (MTOW). Take-off is one of the most critical phases on a flight [9] and to do this safety phase actual airplane weight must be less than the filed length limit weight, the limits for take-off climb, the obstacle clearance tire speed and brake energy capability [10].

Therefore an investigation was performed by the University of San Buenaventura Bogotá and with the support of Copa Airlines Colombia Engineer Operation Area, in order to estimate the theoretical impact of air pollution on the performance of a Boeing 737-800, focusing especially on determining the possible effects on the available payload, since it is a vital variable for safety, operation, and income in commercial aviation [11]. We choose the Boeing 737-800 because Copa Airlines currently has 68 aircraft to carry out its operations and is the model with the largest number of units in this company [12]. We also are going to identify and quantify the impact of all the atmospheric parameters on the air density with the equation of CIPM 2007 [13], which is one of the most important variables to do the take-off calculations [14].

To achieved these aims, we developed a computational application in Matlab to do all the aerodynamic performance calculations, including all the pollution variables. This application was performed using a lot of concepts shown in the book by Holly More [15]. This investigation would have certain limitations like; the aircraft that we developed the study is the Boeing 737-800, the take-off was studied from the head of the runway 13R of the El Dorado airport until the aircraft achieved 800 ft. above the terrain, the engines doesn't have any kind of derate and also isn't included the analysis of the influence of the pollution on the thermodynamic cycle and the combustion, and the runway is always dry during the study.

## 2 | MATERIAL AND METHODS

### 2.1 | Acquisition of atmospheric parameters

To obtain the atmospheric parameters we used a tool from the Bogotá district environment secretary [16] called Bogotá Air Quality Monitoring Network or RMCAB, [17]. From this platform, we analyzed 804 days, two daily measurements. That is equivalent to about 100.000 data [18]. We made the average per year and for each gas that is measured. This information is shown in Table 1.

The average concentration of carbon dioxide in 2018 was 408.22 ppm and at 2019 was from 411.4 ppm [19]. We can see that there were increases between those years. In addition, we can also see that the global average concentration of $CO_2$ is lower than the data from RMCAB shown on Table 1. There is a global concern about the $CO_2$ because is the main responsible for global warming and also is increasing year after year [20].

There are other gases that are not in the RMCAB. For those gases, we are going to use the values assigned by Picard [13].

**TABLE 1** ANNUAL AVERAGE FROM THE ATMOSPHERIC PARAMETERS MEASURE BY RMCAB

| Atmospheric parameter | 2018 | 2019 | 2020 | Average | Units |
|---|---|---|---|---|---|
| $CO_2$ | 427.000 | 423.500 | 420.000 | 423.500 | ppm |
| $CO$ | 0.738 | 0.813 | 0.725 | 0.758 | ppm |
| $O_3$ | 11.317 | 12.350 | 15.883 | 13.183 | ppb |
| $NO$ | 27.175 | 27.175 | 23.463 | 25.938 | ppb |
| $NO_2$ | 17.738 | 17.850 | 19.063 | 18.217 | ppb |
| $NOX$ | 45.113 | 42.913 | 42.488 | 43.504 | ppb |
| $SO_2$ | 1.911 | 2.104 | 2.024 | 2.013 | ppb |
| $HR$ | 64.750 | 64.875 | 62.875 | 64.167 | % |

### 2.2 | Calculation of the variation on the density at Bogotá

In order to obtain a successful approach in Bogotá's air density, it is necessary to take into account that this analysis was carried out from El Dorado airport up to 25000 ft from the head of 13 right runaway, also it is needed the airports elevation above sea level, the air temperature and the pollutants concentration present on the airport surrounding.

Therefore to calculate Bogotá's air density, the pollutants, and other atmospheric parameters must be calculated, beginning with the air pressure in the city that is calculated with the Eq. (1), for this, is required El Dorado's airport elevation (8360ft) [21], and the standard temperature and pressure at sea level

$$P[Pa] = P_o\left(1 - \frac{\lambda \cdot h}{T}\right)^{\frac{G}{\lambda \cdot R_1}} \tag{1}$$

where $P_o$ is the sea level pressure measured in Pascals, T is the sea level temperature measured in $^oC$, $G$ refers to gravity in $m/s^2$, $\lambda$ is a constant with a value of 0.0065, h is the airport height in ft and $R$ is the molar gas constant corresponding to 8.314 J $mol^{-1}$ $K^{-1}$.

As follows, it is needed to find the correction factor due to relative humidity with the Eq. (2)

$$f(P,t) = a + \beta \cdot P + \gamma \cdot t^2 \tag{2}$$

where is $a$ constant equal to 1.00062 $\beta$, is a constant equal to $3.14 \cdot 10^{-8} Pa^{-1}$, P is the pressure at El Dorado airport measured in Pascals $\gamma$, is a constant equal to $5.6 \cdot 10^{-7} K^{-2}$ and t is the outside temperature in $^oC$. The next step is to calculate the saturation pressure of water vapor with the Eq. (3)

$$p_{sv}(t) = exp\left(AT^2 + BT + C + \frac{D}{T}\right) \tag{3}$$

where A is a constant equal to $1.2378847 \cdot 10^{-5} K^{-2}$, T is the outside temperature in K, and finally $B$, $C$ and $D$ are constants equal to $-1.9121316 \cdot 10^{-2} K^{-1}, 33.93711047, -6.3431645 \cdot 10^{3} K$ respectively. The final result is the saturation pressure in Pascal. Then, proceed to find the air molar fraction with the Eq. (4)

$$\chi_v = HR \cdot f(P,t) \cdot p_{sv}(t) \cdot P^{-1} \tag{4}$$

where $HR$ is the relative humidity, and it is expressed in %. After that, we proceed to calculate the compressibility factor $(Z)$ with the Eq. (5)

$$Z = 1 - \frac{P}{T}[(a_o + a_1 t + a_2 t^2) + (b_o + b_1 t)\chi_v + (c_o + c_1 t)\chi_v^2] + \frac{P^2}{T^2}(d + e\chi_v^2) \tag{5}$$

Where $a_o$ is equal to $1,58123 \cdot 10^{-6} K \cdot Pa^{-1}, a_1$ is equal to $-2,9331 \cdot 10^{-8} Pa^{-1}$ and $a_2$ is equal to $1,1043 \cdot 10^{-10} Pa^{-1} K^{-1}$. T refers to the outside temperature in K. $b_o$ has a value of $5,707 \cdot 10^{-6} K \cdot Pa^{-1}, b_1$ has a value of $-2,051 \cdot 10^{-8} Pa^{-1}$, P refers to the atmospheric pressure in Pascals, $C_o$ is equal to $1,9898 \cdot 10^{-4} K \cdot Pa^{-1}, c_1$ is equal to $-2,376 \cdot 10^{-6} Pa^{-1}$,t refers to the outside temperature in $^oC$, $d$ is equal to and is equal to$1,83 \cdot 10^{-11} K^2 \cdot Pa^{-2}$ and $e$ is equal to $-0,765 \cdot 10^{-8} K^2 \cdot Pa^{-2}$. Then we calculate the molar mass of dry air using the Eq. (2.2)

$$Ma = \frac{1}{\sum_{i=1}^{n} x_i}\left(\sum_{i=1}^{n} x_i M_i\right)$$

(6)

where $M_i$ is the molar mass and $x_i$ is the molar fraction (Both are expressed in g/mol) and n are the number of species that affect the molar mass of dry air. Some of the species are $N_2$ , $O_2$ , Ar, $CO_2$ , Ne, He, $CH_4$ , Kr, $H_2$ , $N_2O$, CO and Xe. Finally, we calculate the air's density with the Eq. (7)

$$\rho_a = P \cdot M_a \cdot \frac{[1 - X_v(1 - \frac{M_v}{M_a})]}{Z \cdot R \cdot T} \tag{7}$$

where $P$ is the pressure in Pa, $T/K$ is the thermodynamic temperature = 273.15 + t/$^oC$ and t/$^oC$ is the air temperature (in this case it was used the mean temperature in Bogotá of 15 $^oC$), $X_v$ corresponds to the mole fraction of water vapor, Ma is the molar mass of dry air expressed in g $mol^{-1}$, $M_v$ is the molar mass of water expressed in g $mol^{-1}$, $Z$ is the compressibility factor and R is the molar gas constant expressed corresponding to 8.314 J $mol{-}1$ $K^{-1}$. This method to determine the air's density is calculated using the recommended equation by the CIPM-2007 [13], whose origin is from P. Giacomo [22]. This method to calculate air's density was also used by the Centro español de metrología (CEM) [23]. Comparing this density result of $0.9017 kg/m^3$ , to the one calculated by ideal gases [24] ($0.9063 kg/m^3$), means that there is a decrement of 0.5% because of the pollutants concentration and relative humidity present in the air, affecting directly the local density, what means a direct change on the planes aerodynamic performance and its maximum take-off weight.

## 2.3 | Maximum take-off distance

In order to develop an application that allows develop all the calculations, the first step is defining the fundamental input parameters, meaning this, the direct and indirect variables that lead the user through an accurate analysis. In this case, those initial parameters depend on the Boeing 737-800 configuration just like the engine sea level thrust, the drag and lift coefficient which its maximum value depends on the flaps set, parameter that could be modified in the application; these values are indicated on Table 2 and were obtained from Flight planning and Performance Manual [25].

**TABLE 2** BOEING 737-800 MAXIMUM LIFT AND DRAG COEFFICIENTS

| Flaps setting | Cl max. | $Cd-\mu * Cl$ |
|---|---|---|
| **1** | 2.10 | 0.0070 |
| **5** | 2.14 | 0.0750 |
| **10** | 2.24 | 0.0795 |
| **15** | 2.33 | 0.0840 |

The $\mu$ coefficient shown on Table 2, the maximum take-off weight and the thrust at sea level shown on Table 3 were taken from Jet Transport Performance Methods [26].

Also, there are other initial parameters that don't refer to the airplane, but it does affect it, its runaway physical characteristics that were taken from the *Aeronáutica Civil* [21], the weight restrictions, and the thrust at sea level. Those parameters are shown on Table 3

**TABLE 3** INITIAL VARIABLES

| Parameter | Value | Units |
|---|---|---|
| TORA | 12,467.00 | Ft. |
| ASDA | 12,664.00 | Ft. |
| TODA | 13,451.00 | Ft. |
| Runway Slope | 0.03 | % |
| MTOW | 190,000.00 | Lb. |
| DEW | 92,000.00 | Lb. |
| Thrust at Sea Level | 26,030.00 | Lb. |

The next step is to calculate the speed of sound with Eq. (8).

$$a = \sqrt{K \cdot R \cdot T} \tag{8}$$

Where K is the adiabatic gas constant expressed in $kg/m^2$, R is the ideal gas constant corresponding to $8.314 Jmol^{-1}K^{-1}$ and T is the temperature in K.

Another important step is the one related with the velocity. There are specific velocities that are important for the calculation of the take-off weight.

The first velocity that must be calculated is the stall velocity with the Eq. (9) [27].

$$V_{stall} = \sqrt{\frac{2 \cdot W}{\rho \cdot Cl_{max} \cdot S_{ref}}} \tag{9}$$

Where $Cl_{max}$ refers to the maximum lift, W means the aircraft weight expressed in N, $S_{ref}$ is the wet wing area specified in $m^2$ and $\rho$ is the air density in $kg/m^3$.

The second speed that we calculate was the rotational velocity. This velocity refers to the moment when the pilot pushes up. It is calculated with the Eq. (10).

$$V_R = 1.1 \cdot V_{stall} \tag{10}$$

The next speed that we calculate was the V2. This is the safe flight speed and is calculated with the Eq. (11).

$$V_2 = 1 \cdot 2 \cdot V_{stall} \tag{11}$$

After that we calculate the thrust using Anderson equations Eq. (12) and Eq. (13)

$$\delta_0 = \left(1 + \frac{\gamma - 1}{2} \cdot M^2\right)^{\frac{\gamma}{\gamma-1}} \cdot \frac{P}{Po} \tag{12}$$

Where P refers to the atmospheric pressure measured in Pascals, $Po$ is the sea level pressure in Pa, M is the Mach number and $\gamma$ is the adiabatic gas constant

$$Th = Th_{sl} \cdot \delta_0 \cdot (1 - 0.49\sqrt{M}) \tag{13}$$

Where the quantities and units are: $Th_sl$ is the engine thrust at sea level measured in N, $\delta_0$ is the engine pressure ratio and M the Mach number.

He said that the thrust vary depending on the airplanes speed and the atmospheric conditions, because of that it is necessary to do an iteration at different Mach numbers, in order to obtain a better result [28]. This iteration was done for each knot for a better accuracy.

Next step is to calculate the weight contribution to the plane's acceleration due to the runaway slope with the Eq. (14).

$$F_s = W \cdot sen\phi \tag{14}$$

Where the quantities and units are; W is the airplane weight measured in N and $\theta$ is the runaway slope expressed in rad.

After that the friction force between the aircrafts tires and the runaway was calculate with the Eq. (15).

$$F_r = W \cdot \mu \tag{15}$$

Where the quantities and units are; W is the airplane weight expressed in N and $\mu$ is the friction coefficient between the specific materials.

Next procedure is to calculate the dynamic pressure due to the aircraft's velocity. That is calculated with the Eq. (16)[29].

$$q = \frac{1}{2}\rho V^2 \tag{16}$$

Where the quantities and units are; $\rho$ air's density in $kg/m^3$ and V is the airplanes instant velocity in m/s.

The followed step consists in calculating the c factor or the drag contribution with the Eq. (17) [25].

$$C = (C_d - \mu C_L) q S_{ref} \tag{17}$$

Where $C_d$ is the drag coefficient, $C_L$ is the lift coefficient, $S_{ref}$ means the wing wet area expressed in $m^2$, q is the dynamic pressure in Pa and $\mu$ refers to the friction coefficient between the tires and the runway. The $S_{ref}$ was taken from the Boeing 737-800 specification [30].

$$acc = \frac{g}{W} \cdot (T_h - F_r - C + F_s) \tag{18}$$

Where g is the gravity in $m/s^2$, W is the aircraft weight in N, $T_h$ is the engine thrust in N, $F_r$ corresponds to the friction force in N, $F_s$ is the weight contribution force in N and C is the drag contribution in N.

The next step is to take into account all the previous parameters; calculate the necessary take-off distance for the aircraft, which its increment changes due to a change in velocity and acceleration using the Eq. (19) [31].

$$x_i = \frac{1}{\frac{(a_{sig}+a)}{2}} \left[ \frac{(V + V_{sig})\Delta V}{2} \right] \tag{19}$$

After that, the sum of all the parameters of $x_i$ is performed. This is shown in the Eq. (20).

$$X_I = \sum_{v=0}^{v=v_r} x_i \tag{20}$$

This is an iterative process beginning at the until Eq. (12) and Eq. (20), that is why it is needed the instant velocity and acceleration for a specific time and the same parameters for the immediately next time step.

After that iteration, when the Boeing 737-800 achieves rotational speed and lift off from ground, the turn radius is calculated, which depends on the stall speed and gravity force, and is calculated with the Eq. (21).

$$r = 6.96 \cdot \frac{V_{stall}^2}{g} \tag{21}$$

Also, the angle which the aircraft perform the maneuver is calculated.

$$\theta = \cos^{-1}\left(1 - \frac{h_{ob}}{r}\right) \tag{22}$$

Where $h_{ob}$ is the obstacle height and R is the turn radius. Both are expressed in m.

The next parameter corresponds to the horizontal distance that the airplane travels during its rotation calculated with the Eq. (23).

$$S_a = r \sin(\theta) \tag{23}$$

The final step is to calculate the take-off distance. The take-off end when the aircraft is 35 ft above the ground. This calculation is made with the Eq. (24).

$$x_T = x_I + S_a \tag{24}$$

## 2.4 | Maximum take-off weight

All the calculation need to accomplish the regulation aviation from the FAA [32].

To calculate the take-off weight is necessary to make an iterative process. This iteration begins with the DEW and requires to compliance the following statements:

1. For the Case 1: Add one pound if the $x_I$ is lower than TORA and the xt is lower than TODA.
2. For the Case 2: Add one pound if the $x_I$ is lower than TORA and the xt is lower than TODA.
3. For the Case 3: Add one pound if the $x_I$ is lower than ASDA.
4. For the Case 4: Add one pound if the $x_I$ is lower than ASDA.
5. The iteration would stop if the calculated weight reaches the MTOW.

### | Case 1

The first case refers to the situation where the aircraft begins its normal take-off run, go through V1, has all its system working normally and take-off with both engines operative, so the whole thrust is available. To do this case we double the result from the Eq. (13), because the aircraft is operating with both engines.

### | Case 2

The second case occurs when the aircrafts begin its normal take-off, go through V1 speed, and presents an engine failure and the pilot decide to continue the take-off with the thrust detriment. If the aircraft can't perform the take-off, then the take-off weight must be less.

To simplify, it is assumed that the engine failure occurs at V1, which is the most critical speed. So, it is necessary to calculate this speed. To perform this calculation we used the V1 data found on the PEM [33]. We made a lineal regression shown on the Eq. (25) with the data that we found on the PEM.

$$V_1 = 0,0006W + 55,818 \tag{25}$$

Where W is the aircraft weight in lb and V1 is the decision speed in knots.

Turbofan engines have a spindown while their revolution decrees until it finally stops. In other words, the engine would produce thrust during a few seconds until it would be completely stopped. One kind of spindown is the fuel cut, which can be assumed like an engine failure. The JTPM shows a graphic for this behavior [26], so with the help from web plot digitizer and excel we made an exponential regression. We choose this type of equation because its asymptote is on cero. The equation that we get is the Eq. (26).

$$Y_1 = 1,5907e^{-1,687x} \tag{26}$$

Where Y1 is the factor that multiplies the thrust from one engine and x is the time since the engine failure occurs expressed in $s$.

After that, and considering the thrust decrement, continue with the case 1 calculations to find V2, the lift off speed, the rotational radius and climb angle in order to find the take-off distance, iterate and finally get to the maximum take-off weight.

### | Case 3

The third case occurs when the aircrafts begin its normal take-off, and before V1 speed presents an engine failure and the pilot decide to abort the maneuver, initializing a brake operation to stop the aircraft before the runaway ASDA ends, during which by regulation the engine reversible aren't available.

In order to put a safety factor, it is assumed that the engine failure occurs at V1, which is the most critical speed, and the aircraft needs a greater distance to fully stop the aircraft. Once it gets to the decision speed, the damage engine thrust decrease according to the fuel cutoff while the other engine in analyzed under the throttle chop engine spindown. To find this behavior we used web plot digitizer and excel to find the following polynomial.

$$Y_2 = -7 \times 10^{-5} X^5 + 0,0026X^4 - 0,0373X^3 + 0,2569X^2 - 0,8502X + 1,2539 \tag{27}$$

Where $Y_2$ is the factor that multiplies the thrust from one engine and x is the time since the throttle chop in $s$.

This equation has an important limitation. This only works during the spindown, then the engines enter in idle condition. For that reason, the Eq. (27) only works for the first 12 seconds after the throttle chop, after that the factor that is going to multiply the engine thrust would be 0.08894.

Once the thrust detriment is calculated, proceed to iterate the distance traveled from the decision speed until the airplane fully stops, this must be performed by decreasing its speed, with a negative acceleration due to braking. This braking contribution considers a 0.38 friction coefficient recommended from Boeing at the JTPM [26].

## | Case 4

The fourth case analyze a take-off abort due to an unforeseen event in one or more of the essential system other than engines, the decision to stop occurs at V1, therefore, the aircraft must break before runaway ASDA ends, without the engines reversible due to safety procedures.

The calculation procedure is like the third case, since the beginning of take-off until the decision speed, where the pilot decides to stop the procedure, in this case the aircraft has all its engines operating normally, so at V1 the pilot realized a throttle chop for both engines and applies brakes in order to stop the airplane and abort safely the mission before runaway ASDA ends.

**Climb limit** The climb limit was calculated with a parametric equation. First we take the graphics from Flight planning and performance manual [25]. After that we made a polynomial regression with the help from web plot digitizer [34] and excel. Finally we made the parametric equation using Thomas principles [35].

$$W = -0.095T^3 - 7.038T^2 - 108.5T + 199,459 - 5h_p \tag{28}$$

Where T is the outside temperature in $^oC$ and $h_p$ is the pressure altitude in ft.

The tire speed limit was made with a lineal regression. We take the graphics from the FPPM [25] and with the help from web plot digitizer [34] and excel, we made the lineal regression shown on the Eq. (29).

$$W_t = -625.725T + 174720 \tag{29}$$

Where T is the outside temperature in $^oC$.

The Eq. (26) and Eq. (27) has one important limitation that they only work with a flaps setting 1.

When the calculations of tire speed or climb limit is greater than the MTOW, then the calculated take-off weight would be the same as the MTOW.

To calculate the obstacle clearance limit, we made the same calculation from the Case 2. Once the aircraft finishes its take-off phase, it begins with the climb until reaching 800 ft of height. Therefore, we take the angle calculated in Case 2 and the distance traveled to that point. Then we perform the calculations of the ascent path. To compare the trajectory with the obstacles present in the city of Bogotá, a linear function will be generated. This lineal function is presented in the Eq. (30).

$$y = mx + b \tag{30}$$

Where y aircraft altitude in ft, m is the aircraft climb angle in rad and b is the constant that is calculated with trigonometry.

To calculate the obstacle clearance height we used the Eq. (31). taken from JTPM [26].

$$h_{o+fc} = h_o + 35ft + x \cdot 0.8\% \tag{31}$$

Where $h_o$ is the obstacle altitude and x is the distance from the runway head to the obstacle. Both are expressed in ft.

The weight iteration from the climb limit is similar to the Case 2. There are two important differences; the first one is that the iteration would end when the calculated take-off weight would be equal to the climb limit take-off weight. The second one is that the aircraft height should be always greeter than the obstacle clearance height.

An example is presented on the Fig. 1. This example was made with an outside temperature of $15^oC$ and with a pressure altitude of 8100 ft. The iteration in this case stoped because the calcualted weight (153,896 pounds) was equal to the climb limit weight.

On the Fig. 1 we can also see that the height obstacles clearance doesn't touch the aircraft trajectory.

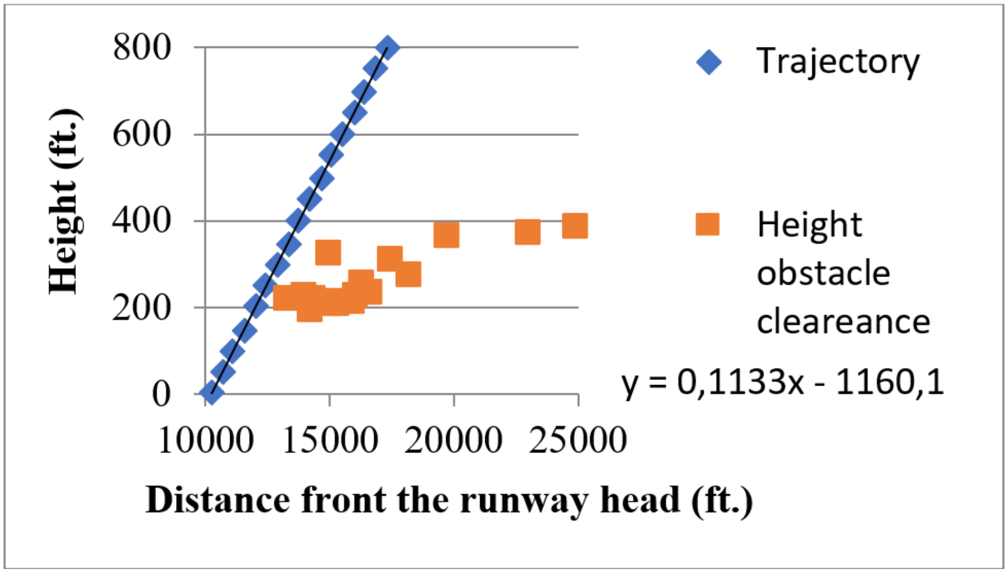

**FIG. 1** Aircraft Climb Path at the end of the iteration

It is important to remember that any aircraft can do a take-off if their weight is more than the most limiting weight calculated according to the AFM [36] and also the FAA [37].

## 3 | RESULTS AND DISCUSSION

### 3.1 | Applicative validation

To validate our application (Aircraft take-off Software ATS) we compare three different events with the program AFM-DPI from Boeing [38]. The program developed seek to incorporate the pollutant levels of the air in the analysis of the initial performance of the aircraft, which are not taken into consideration in the AFM-DPI program from Boeing. Our program would be validated if the error percentage is less than 1%.

On Table 4 we can see that the error percentage is lower than 1% in each of the events that were evaluated. The maximum error found was from 0.869 %. These errors appear because the equation's regression is not completely accurate. All of them have a small percentage of error.

We can also see that the climb limit is equal to the obstacle clearance limit in both applications. That suggests that the obstacle clearance limit is limited only by the climb capability, in other words, the obstacles are too low to limit the take-off weight.

Another important observation is that in the application ATS the most limited weight from the filed length limit was always the Case 2. This weight is presented in Table 4 and was the weight used in the validation. So that means that the Case 1, 3, and 4 was not available to perform the validation because the AFM-DPI does not show those results.

**TABLE 4** INITIAL VARIABLES

| Event | Cases | W (AFM-DPI) (lb.) | W (ATS) (lb.) | Error (%) |
|---|---|---|---|---|
| At 15°C and 8100 ft. | Field length limit | 161,529 | 162,049 | 0.321 |
| | Climb limit | 154,007 | 153,686 | 0.209 |
| | Tire speed limit | 164,799 | 164,256 | 0.331 |
| | Obstacle clearance limit | 154,007 | 153,686 | 0.209 |
| At 14°C and 8360 ft. | Field length limit | 160,51 | 159,127 | 0.869 |
| | Climb limit | 152,916 | 152,795 | 0.079 |
| | Tire speed limit | 163,649 | 164,877 | 0.745 |
| | Obstacle clearance limit | 152,916 | 152,795 | 0.079 |
| At 21°C and 8100 ft. | Field length limit | 158,623 | 159,006 | 0.241 |
| | Climb limit | 151,142 | 151,19 | 0.032 |
| | Tire speed limit | 161,191 | 160,526 | 0.414 |
| | Obstacle clearance limit | 151,142 | 151,19 | 0.032 |

### 3.2 | Analysis of the pollution and the relative humidity impact on the air's density.

Density is calculated through two different methods. The first one is the ideals gas method, that was used just to validate the applicative, where it was found that in a range between 5 and 25 degrees Celsius, a change in one degree generate an increase or decrease of about $0.034kg/m^3$.

The second method is using the CIPM-2007 Eq. (13). This method is used to recalculate the air's density with the proportional influence due to atmospheric composition, pollutants, and relative humidity. The data was taken from RMCAB which averaged are on Table 1, making this analysis more accurate and useful.

The average pollution in Bogotá's air is different to the air's standard pollution concentration, therefore using the average on Table 1 , the air density increased in 0.0012%. One of these components, Carbon dioxide $(CO_2)$, has the highest concentration from all the pollutants gases available on the RMCAB study and is the one that most affects air density, not only for its concentration but also for its contribution on air density.

We found the minimum and the maximum pollution concentration from all the time that was studied. These results are shown on Table 5.

Leaving pollutants aside, the relative humidity is another important and very decisive variable for calculating the actual air density, because the equation selected for the calculations considers the amount of water vapor present in Bogotá's air, which has an average relative humidity of 64.16% from data measured by RMCAB. When we used this percentage of relative humidity, considering the pollution concentration average, the air density would decrease 0.5523% from the calculated without the presence of water vapor. This calculation

**TABLE 5** MAXIMUM AND MINIMUM CONCENTRATION ON THE AIR'S GASES FROM THE RMCAB

| Atmospheric parameter | Minimum | Maximum | Units |
|---|---|---|---|
| $CO_2$ | 294.00 | 575.00 | ppm |
| $CO$ | 0.01 | 3.40 | ppm |
| $O_3$ | 0.50 | 85.00 | ppb |
| $NO$ | 0.01 | 199.10 | ppb |
| $NO_2$ | 0.80 | 58.00 | ppb |
| $NOX$ | 2.00 | 300.80 | ppb |
| $SO_2$ | 0.01 | 23.50 | ppb |
| $HR$ | 31 | 100 | % |

was always done with a constant outside air temperature of $15^oC$.

In the same way, with a higher temperature the rate of density decrease will be greater, while if the temperature decrees, the density drop will be attenuated as well. For example, at $5^oC$ and considering an 80% of relative humidity the density decrees in 0.2818%, meanwhile with the same relative humidity but this time at $25^oC$, this reduction is 1.0335% from to the calculated without water vapor.

### 3.3 | Analysis on the maximum take-off weight

Once the validation of the weights and the analysis of the behavior of the density in Bogotá were carried out, the impact of the pollutants was directly analyzed on the take-off weight. The contribution generated by each atmospheric parameter to the weight of the aircraft is given by Table 6, which was performed with the initial variables of the third case study previously analyzed: a temperature of $21^oC$ and a pressure altitude of 8100 ft; because in this case the smallest error was observed in the validation process.

**TABLE 6** POLLUTANTS INFLUENCE ON THE take-off WEIGHT.

| Atmospheric parameter | Standard RMCAB concentration (ppm) | Contribution to the take-off Weight (ppm/lb.) | |
|---|---|---|---|
| | | Case 2 | Climb limit |
| $N_2$ | 780,848.000 | -192.30 | -238.09 |
| $O_2$ | 209,332.000 | 60.97 | 74.62 |
| $Ar$ | 9,332.000 | 16.66 | 20.41 |
| $CO_2$ | 423.500 | 12.50 | 16.60 |
| $Ne$ | 18.200 | -20.25 | -27.00 |
| $He$ | 5.200 | -7.14 | -8.33 |
| $CH_4$ | 1.500 | -19.80 | -19.80 |
| $Kr$ | 1.100 | 3.41 | 4.13 |
| $CO$ | 0.758 | -200.00 | -200.00 |
| $H_2$ | 0.500 | -6.80 | -8.26 |
| $N_2O$ | 0.300 | 12.19 | 14.93 |
| $Xe$ | 0.100 | 1.78 | 2.19 |
| $NOX$ | 0.044 | 100.00 | 100.00 |
| $NO$ | 0.026 | 200.00 | 250.00 |
| $NO_2$ | 0.018 | 10.75 | 13.15 |
| $O3$ | 0.013 | 9.61 | 11.90 |
| $SO_2$ | 0.002 | 5.21 | 6.41 |

The contribution to the take-off weight is shown in Table 6. Is also showed the required amount of particles per million to increase the take-off weight in one pound. The negative signs ("-") symbolizes, that when this atmospheric parameter decreases then the take-off weight increases. The first Case was not analyzed because this case does not limit the take-off weight. In other words, the calculated take-off weight in this case is equal to the maximum take-off weight. On the other hand, the Case 3 and 4 does not evidence an increase of one pound. From Table 5 and taking into account the maximum and the minimum concentrations shown on Table 4 we can conclude that in Bogotá the carbon dioxide $CO_2$ is the only pollution gas that has a significant influence on the take-off weight. The other pollution gases like CO, $NO_X$, $NO$, $NO_2$, $O_2$ and $SO_2$ require a bigger increase than the maximum or the minimum founded in Bogotá. For that reason, the contribution from those gases is insignificant.

We also should consider the relative humidity. This atmospheric parameter has a significant influence in the Case 2, 3 and climb limit. The relative humidity contribution is shown on Table 7.

**TABLE 7** RELATIVE HUMIDITY INFLUENCE ON THE TAKE-OFF WEIGHT

| Atmospheric parameter | Contribution to Aircraft Weight (pmm/lb) | | |
|---|---|---|---|
| | Case 2 | Case 3 | Climb limit |
| Relative humidity | -0.026 | -0.0598 | -0.0657 |

The impact of relative humidity is significant in the cases evidenced in Table 7. For example, in Case 2 with an increase of 0.01488% in relative humidity, the take-off weight is decreased by one pound. That means that if the relative humidity reaches its maximum percentage (100%), the decrease in weight can reach 3839 lb for case 2, 1624 lb for the case of ascent and 1667 lb for case 3. However, it should be noted that this increase is obtained by keeping the other variables constant, which does not occur in real terms, since an increase in humidity would directly affect the other parameters already analyzed.

Following this analysis, the environmental impact study of the three events was carried out. Comparing the standard atmospheric parameters with the average atmospheric parameters of Bogotá through the ATS application.

**TABLE 8** COMPARISON BETWEEN THE STANDARD ATMOSPHERIC PARAMETERS WITH THE AVERAGE ATMOSPHERIC PARAMETERS

| Event | Cases | ATS Standard (lb.) | ATS average (lb.) | Difference (lb.) |
|---|---|---|---|---|
| At 15°C and 8100 ft. | Case 2 | 162,049 | 159,344 | 2,705 |
| | Case 3 | 185,114 | 180,304 | 4,810 |
| | Climb limit | 153,683 | 152,971 | 712 |
| At 14°C and 8360 ft. | Case 2 | 159,127 | 158,299 | 828 |
| | Case 3 | 183,637 | 183,637 | 0 |
| | Climb limit | 152,795 | 152,123 | 672 |
| At 21°C and 8100 ft. | Case 2 | 158,494 | 155,81 | 2,684 |
| | Case 3 | 185,304 | 183,959 | 1,345 |
| | Climb limit | 150,773 | 149,733 | 1.040 |

From Table 8 it can be seen that the most limiting weight in all the events analyzed is the climb limit. This weight varies between 1040 and 672 pounds less. So, including the average atmospheric parameters to the aerodynamic performance calculation for the take-off, limited the take-off weight in Bogotá.

## 4 | CONCLUSIONS

- The inclusion of the atmospheric parameters, especially the increase of the relative humidity on the calculation significantly decreases the aircraft MTOW up to 2.66%, equivalent to 2181 kg.
- The take-off weight is proportional to the carbon dioxide $C0_2$ concentration, while the relationship between the take-off weight and the relative humidity is inversely proportional being the last one the most significant on the weight calculations. The relative humidity can modify the take-off weight between 709 kg and 1811 kg on a Boeing 737-800.
- The most limiting weight for the aircraft Boeing 737-800 taking off from the 13R runway in El Dorado airport, is the capability to climb.
- Case 2 (engine failure during take-off) presented the greatest limitation with respect to the maximum weight per runway length of all the cases. On the other hand, the Case 1 (normal take-off with 15% of safety factor) doesn't show any kind of limitations. On this case the calculations shows that the take-off weight is equal to the MTOW.
- Carbon dioxide $(CO_2)$ is the only quantifiable pollutant that has significant implications in the maximum take-off weight form Bogotá. Its average concentration due to the aerodynamic effect generates an increment in the maximum take-off weight. However, this gas is also related to the relative humidity and considering both parameters generate a decrease of approximately 400 kg in the maximum take-off weight.
- The rest of *the gases* measured from the used monitoring tool (RMCAB), does not affect the take-off weight. It is recommended for further investigation to carry out a study of those other gases that weren't quantified through the present project to know if the variation in the concentration for those gases affects the take-off weight.
- The average pollutants concentration and the relative humidity that compound the Bogotá's dry air molar mass shows an increase of 0.0006% compared to the standard concentrations determined by Picard [13]. As a consequence of this increase in the molar mass, there is an increment of 0.0012% in the Bogotá's air density.
- It was developed and validated an application called Aircraft take-off Software (ATS) that considers the pollutants present on air and some other environmental parameters, to calculate the maximum take-off weights of a Boeing 737-800 to do a take-off from the 13R runway at El Dorado airport.
- The analysis developed through this project was centered on the aerodynamic performance impact on a B737-800 due to the environmental conditions at El Dorado airport, and how it affects the aircraft maximum take-off weight. However, combustion and engine degradation weren't analyzed in this document, for further investigation, it is recommended to analyze how the engine performance is affected by the pollutants in the air that is passing through.
- A mathematical model was created to analyze conceptually the Boeing 737-800 aerodynamic performance and to simulate different situations the airplane could experiment during take-off from Bogotá, regarding its atmospheric parameters and pollutants concentration.
- The deterministic mathematical model allows optimizing and recreating a suitable take-off performance simulation considering the air pollutants levels presents over El Dorado airport, and its operational affectation during take-off.

### Declaration of Interest

The authors declare that there is no conflict of interest.

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